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PHYSICAL PROPERTIES AND THE MECHANICS OF PENETRATION AND LOADING OF BULK MATERIALS AS RELATED TO RAPID EXCAVATION

Kelvin K. Wu, et al

Bureau of Mines

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Physical Properties and the Mechanics of Penetration and Loading of Bulk Materials as Related to Rapid Excavation

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13. ABSTRACT

Basic physical properties tests on 38 samples from tunneling and mining sites were completed. Designing and constructing a curved path penetration and loading apparatus has been accomplished. Linear penetration tests were run using both confined and unconfined materials. Confined tests were performed with bulk material in the loose state and then on a vibratory compacted state. Compaction was found to be the most important variable. Digging and loading tests indicated the tremendous energy required for the initial digging process compared to the loading process.

Load cells were installed on the walls of the containers to measure the force transmitted to the walls during penetration tests. The theoretical penetration curves constructed through mathematical analysis generally fit well with the experimental curves, but the loading curves are not satisfactory. This was mainly caused by the design of the wedges. A finite element computer program was completed.

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4800 Forbes Avenue
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Principal Investigator: Dr. Kelvin K. Wu
Telephone No.: 412-621-4446

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ANNUAL TECHNICAL REPORT SUMMARY

Objective

To determine the effects of physical properties of materials fragmented during the rapid excavation process on the equipment response, and to analyze the mechanics of flow and the machine--material interface during penetration and loading.

Research Plan

Sample acquisition and basic physical properties measurements constitute the first stage of work before any investigation of the mechanics of penetration and loading. Experimental tests in the laboratory on penetration and loading follow subsequently at first and then can be carried on concomitantly with the property measurements. The theoretical analysis of the mechanics of flow and loading of representative bulk materials enable establishing the necessary mathematical models. Plasticity theory and the finite element method are utilized in this phase of the work.

Major Accomplishments

Thirty-eight samples have been obtained from tunnel and mining sites. Basic physical properties tests on these bulk materials were completed. Designing and constructing a curved path penetration and loading apparatus has been accomplished.

Linear penetration tests were run using both confined and unconfined materials. Confined tests were performed with bulk material in the loose state and then on a vibratory compacted state. Compaction was found to be the most important variable. Digging and loading tests clearly indicated the tremendous energy required for the initial digging process compared to the loading process.

Since the materials encountered during the test were very irregular, the penetration and loading curves are generally noisy. A specially designed electronic integrator was used to overcome this problem. Smooth curves of work as a function of depth of penetration were obtained upon integration. Regression equations were then determined. Force and energy per unit volume can be calculated also from these equations (table 1).

Load cells were installed on the walls of the containers to measure the force transmitted to the walls during penetration tests. The theoretical penetration curves constructed through mathematical analysis generally fit well with the experimental curves, but the loading curves are not satisfactory. This was mainly caused by the design of the wedges. A finite element computer program was completed.

TABLE 1. - Equations for Work as a Function of Depth of Penetration (Work = K_1 Depth + K_2 Depth^2) for Various Wedges and Minus 2-inch Copper Ore

State	Unit	Wedge type	Wedge angle, degree	K_1	K_2	Total work ft-16
L	112	R	45	-2.75	2.15	61
CM	126	R	45	-4.81	4.97	150
L	56	R	30	-0.96	1.00	30
CM	83	R	30	-5.55	3.65	99
L	53	R	20	3.34	0.25	29
CM	119	R	20	-12.7	6.07	142
L	50	R. C.	45	2.38	0.36	27
CM	119	R. C.	45	0.006	3.95	142
L	103	R. C.	30	-2.09	1.90	56
CM	116	R. C.	30	-2.01	4.16	138
L	129	R. C.	20	2.80	1.48	70
CM	132	R. C.	20	-8.60	4.47	157
L	80	H	45	-2.01	1.53	43
CM	156	H	45	-5.99	6.17	186
L	54	H	30	-0.98	0.97	29
CM	126	H	30	-6.03	5.17	150
L	58	H	20	1.71	0.60	32
CM	81	H	20	-3.17	3.19	96
L	27	C	45	1.19	0.22	15
CM	87	C	45	-7.43	4.13	104
L	29	C	30	-0.30	0.49	16
CM	105	C	30	-5.40	4.38	125
L	22	C	20	0.82	0.20	12
CM	75	C	20	-3.96	3.13	89
L	69	C	15	1.67	0.776	38
CM	37	C	15	-3.42	1.80	44

L: Loose; CM: Compact; R: Full Wedge; H: Half-Wedge; C: Conical Wedge; R.C.: Full Wedge with Circle Rod.

INTRODUCTION

In the mining industry, the rate of loading and conveying of mined material from the working area all the way to a storage or shipping area is a factor which affects the rate of advance of excavation and can be a bottleneck. This is especially true as the rate of advance in hard rock using conventional fragmentation methods increases significantly.

The principle objective of the research at the Pittsburgh Mining and Safety Research Center for materials handling is to find out what factors influence the penetration and loading processes and what effects the physical properties of the mined materials have on these processes. Through these research efforts, it may be possible to obtain better means for selecting handling equipment and an efficient method of removing mined materials.

TECHNICAL DISCUSSION

The emphasis in materials handling during excavation and loading has been on the design and manufacture of larger and more complex mining equipment with very little research on the physical properties of the bulk materials and the mechanics of penetration and loading. Experience has been used primarily since there is a lack of a rational equipment selection procedure based on first principles and the mechanics of excavation-loading processes.

Operations research techniques are certainly of great utility in systematizing the analyses of existing and proposed equipment combinations and even single units through the use of time-study data and deterministic concepts. The latter has particular appeal because of its predicative potential with respect to new situations in contrast to probabilistic based simulations of existing systems. However, the utility of the output of computer implemented models of mining systems, as with any computer program, is constrained by the reliability of input - in the vernacular, garbage in-garbage out. It is therefore well to keep in mind that the materials handling system in any mining operation begins at the business end of the excavating machine, that is, at the working edge of the digging tool.

The force a machine is capable of applying to the digging teeth and the response of the material to the penetrating tools (if penetration occurs) will determine whether or not the machine is capable of excavating a given material. Examination of the digging tools of diverse mining equipment, coal ploughs, backhoes, scrapers, bucket ladder dredges, certain types of drills and so forth, shows that many have in common wedge-shaped digging teeth or edges.

One is thus led to an examination of wedge penetration processes in geologic materials as a rational basis for selecting certain types of mining equipment.

The assumption (or presumption) is that the equipment manufacturer knows the force capability of his machine (otherwise how would one design the machine). It is then a question of first determining the material response to load and second of efficiently matching the force capability of the machine to the penetration response of the material (analogous to the matching of pump or fan capabilities to pipe or duct responses. Thus, fundamental equipment selection parameters emerge: machine force and material response.[1]

It is essential to the validity of the foregoing chain of thoughts, of course, that digging forces do in fact constitute a major aspect of machine performance. That this is so is clearly the case for bucket wheel excavators, as an example will show, and it seems likely to be the case for numerous other equipment.

Consider a bucket wheel excavator (BWE) operating in ground that in "diggability" lies between hard rock and that must be blasted and mined rock in a stockpile. Typically BWE's are high capital cost, high volume production units. Operating costs are mainly those of power and maintenance. Of the total power requirements, approximately 90 percent will be expended in digging. One formula for computing digging power is:

$$P = \frac{K\sqrt{QRS}}{c\eta} \qquad (1)$$

where:

P = digging power (KW)
K = specific cutting resistance (kg/cm)
Q = digging capacity (m^3/hr)
R = radius of the cutting wheel (m)
S = bucket discharges per minute (./min)
η = drive efficiency
c = a digging parameter (c=171 for a bench height to wheel diameter ratio of 2/3)

According to formula (1), operating power cost of a BWE is a linear function of the force-penetration characteristic (specific cutting resistance) of the material being excavated.

It is important to note the assumption implicit in formula (1) that the material is in fact being dug and full buckets are obtained. If the force-penetration characteristic of the material is not properly matched to the machine capability, poor digging action will result; design production will

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[1] The physical problem is mechanical so that an integration of the force-penetration function yields a work-penetration function. Division by the volume of the displaced material results in a work per unit volume-penetration characteristic which is preferred by some as a measure of "diggability". In a broad sense, the two are seen to be synonymous, one derivable from the other.

not be met. Costs will rise; wheel speed probably will be increased and only partially filled buckets obtained. The entire operation may be placed in jeopardy; a multi-million fiasco becomes an all too real possibility that could have been avoided by closer attention to first principles and fundamentals of excavation-penetration processes.

A rational equipment selection procedure based in part on the matching of equipment force capability to material response characteristic thus seems worthy of consideration. Obviously not all conceivable mining systems will be competitive under a prescribed set of circumstances. Other selection criteria and design constraints will eliminate certain types of equipment from consideration at the outset. Among competitive systems, however, machine force and material response must rationally weigh heavily in the selection-system design process.

Although diverse in appearance, the basic function of excavating equipment is to dig. And although excavator useage occurs under a wide range of environmental conditions, the digging function is in the majority of cases accomplished essentially through processes of penetration by wedge and cone-shaped teeth. The former shape is predominant.

Two classification schemes for penetration processes in geologic materials that have proved useful are based upon the nature of the material penetrated and upon the type of trajectory available to the penetrating tool. The schemes are arbitrary but do serve to bring order to a sizeable array of superficially unrelated field situations.

The interaction of strength and grain size throughout the spectrum of geologic materials - rocks and soils - results in two distinct types of force-penetration functions, for a given wedge tooth, and enable one to classify the penetration process as one occurring in a continuum or discontinuum. If the ratio of the length (L) of the cutting edge of the wedge to the diameter (D) of the average grain size is large, then the penetration process occurs in a continuum; if the ratio (L/D) is moderate then the process occurs in a discontinuum,. and if L/D is small the continuum case again occurs. An order-of-magnitude argument suffices to quantify the meaning of large, moderate and small for the purpose at hand. Thus, if

$L/D > 10$ penetration occurs in an infinite continuum
$10 > L/D > 1$ penetration occurs in a discontinuum
$1 > L/D$ penetration occurs in a finite continuum

Penetration of an infinite continuum requires overcoming of the strength of the particle aggregate constituting the medium; penetration of a finite continuum is essentially a single particle penetration process and therefore requires overcoming the strength of an individual particle. Penetration of discontinua involves both requirements as reflected in a pushing-aside action on individual particles and single particle splitting. The later type of event may occur at the limit of embedment during a head-on encounter between digging tool and particle.

In practice, however, single particle splitting is rare due to rotation of the particle off the penetration path and to machine stall. Typically penetration of continua is associated with a smooth force-penetration function whereas penetration of discontinua is characterized by a "noisy" force-penetration function.

From the standpoint of the penetrating tool, penetration processes are at one extreme fixed path processes in which the digging tool is constrained to move along a prescribed path, for example in the case of a bucket wheel tooth. At the other extreme, free path processes are identifiable as in the case of projectile or ballistic penetration. Excavation penetration processes are more akin to the former than to the latter. Operator control of the machine, however, permits modification of the digging tool path in most instances (and thus avoids stall), so that an element of the free path process occurs in what are basically fixed path processes.

Ballistic or free path penetration is also a dynamic process requiring explicit recognition of inertia forces. The problem is a complicated one with a long history of interest and experimentation. Fixed path processes, which are more the rule than the exception in the mining industry, are less the impact-deceleration type and more the sustained load type of penetration process.

An idealization appropriate for detailed study of excavator penetration mechanics is thus one of a wedge shaped tool penetrating geologic materials quasi-statically along a prescribed path.

The important technologic aspects of penetration processes has resulted in series of analytical, numerical and experimental investigations of idealized wedge penetration problems within the context of plasticity theory. In plasticity theory the actual material is replaced by an idealized continuum that deforms elastically up to some state of stress at which large, permanent deformation and failure may occur. The state of stress at failure is given by the yield function. Other essential elements of theory are the equations of equilibrium and the geometry of strain, which unlike the yield condition are independent of the material, and the constitutive equations. The latter are usually obtained from the yield condition through the principle of normality. The yield condition or failure criterion plays a central role in plasticity theory. The central aim of a plasticity theory is the description of the mechanical response to load materials deforming beyond the range of purely elastic behavior which must be the case if penetration and therefore excavation are to occur.

Quasi-static wedge penetration of isotropic rocks and soils has been investigated rather extensively, and experimental evidence tends to substantiate the theory as regards the prediction of force-penetration functions. Inputs to the general problem include specification of the wedge geometry, material strength, interface friction and loads on the material adjacent to the wedge shoulder. The main results of the solution

to the general problem are: (1) the force penetration function is linear in the case of strong media (body forces negligible) and (2) the work per unit volume of displaced material is a constant - a characteristic of the material. Thus if F is the force required for penetration per unit length of cutting edge to a depth h, then

$$(1) \qquad F/h = K_1 \qquad \text{(strong media)}$$

and if W is the work of penetration per unit width of cutting edge to depth h requiring displacement of volume V of material then

$$(2) \qquad W/V = K_2 \qquad \text{(strong media)}$$

The parameters K_1 and K_2 of formulas (1) and (2) which are derivable one from the other characterize the material response to digging tool loads and can be computed from simple field or laboratory tests in advance of actual mining operations. Values of K_1 are of the order 10^5 (lbs/in) for strong rock and range down to less than 10 (lbs/in) for weak, unconsolidated soils. The five orders of magnitude range of K_1 certifies its sensitivity as a diggiability design parameter. Moreover K_1 in the dynamic case does not appear to differ significantly from static values. However, the energy per unit volume of displaced material in the dynamic case is commonly twice the work per unit volume (K_2) required for penetration in the static case. In the dynamic case, the kinetic energy of the digging tool at the moment of contact with the material is taken as the energy expended in excavation.

In the case of weak "soil", body forces are no longer negligible, consequently the force-penetration characteristic is no longer linear. However, good approximations to the analogues of (1) and (2) are

$$(3) \qquad F/h^2 = K_1 \qquad \text{(weak media)}$$

$$(4) \qquad W/V = K_2 h \qquad \text{(weak media)}$$

which is to say that the force-penetration characteristic for soil is parabolic and that the work per unit volume of excavation increases linearly with depth.

As the penetration process continues beyond initial indentation, the relationships between force and penetration depth become complex. A helpful view of the subsequent "deep" penetration process is to suppose that the penetration force is composed of forces on the digging tool and tool mount. The sum of the loads developed on the wedge tooth and "shaft" behind the tool then constitute the total force required for continued penetration. The latter is a frictional drag type of force and will be determined by the increase of "shaft" area and shear stresses with depth. In fact, the "shaft" may be the bucket proper in the case of a BWE. The tooth load will be similar to that required for initial indentation but also modified by the variation of stress with

depth. In the majority of penetration-excavation processes it would appear that tooth loads are predominant (compared to frictional drag on the buckets). Moreover, curved penetration paths are the rule, so that "deep" penetration-excavation processes are more akin to "machining" than, say, to "pile-driving" processes. For example, bucket wheel excavation may be considered as a process of "machining" ground from the parent mass by means of a rotating, toothed cutter. This idealization also lends itself to a detailed mathematical analysis based on first principles within the context of plasticity theory. Indeed, a large variety of practical penetration-excavation processes are amenable to such treatment.

Penetration of finite continua ($1 \gg L/D$), such as single blocks, boulders and pebbles, is of necessity accompanied by splitting of the particle. In practice, an encounter between a digging tooth and a hard boulder embedded in a matrix of weak or unconsolidated material is typified by the development of large impact loads on the machine and perhaps sudden machine stall. If stall is to be prevented, then the particle must either split or rotate off the penetration path. The point load P (lbs) required to split a particle of characteristic linear dimension $\underline{d}$ (in) is proportional to the product of tensile strength to (psi) and d-squared, thus

$$P \propto \Gamma_o d^2 \quad \text{(single particle splitting)} \tag{4}$$

If the centroid of the particle does not lie on the penetration path, tooth-particle contact will be eccentric. The tooth load required to rotate the particle clear of the tooth edge depends upon the firmness of particle embedment, that is, upon the in situ state of stress about the particle. If embedment is strong, machine stall may again occur. However, if the eccentricity of contact is large, point contact is not possible. The penetration load is then due to shoulder contact.

Tensile strength of a large number of rocks is of the order 10^3 (as determined by the Brazil test), so that P is approximately 1,000 times the square of the particle diameter. If an excavating machine is designed to penetrate weak geologic media that require forces of the order of 10 pounds per length of tooth cutting edge, then a splitting collision with a 10-foot-diameter boulder would result in a design overload of approximately 10^5. Clearly the machine hazard represented by buried boulders in soft ground is substantial. The size and frequency of such boulders in a deposit or pile of otherwise "easy" diggability may seriously impair equipment performance. Rational loading and excavation equipment selection should therefore quantitatively anticipate and assess such conditions. Large rocks in fragmented materials obtained during rapid excavation present similar problems although not of such a magnitude.

As the number of large rocks embedded in the material increases, the medium becomes less a continuum and more a discontinuum. In the range of ratios of tooth length (L) to particle diameter (D) between one

and ten, the force-penetration function becomes quite "noisy," and the smoothness of the continuum force-penetration function is lost. No well developed theory appears to be available for a detailed analysis of discontinuum penetration processes at this time. However, exploratory experimentation has substantiated several aspects of a working hypothesis concerning the penetration of discontinua by wedge shaped digging tools.

A fundamental postulate concerning the mechanics of discontinua is that the amplitude of fluctuations in the penetration force during penetration becomes large and that the amplitude and frequency of such fluctuations are functions of particle size. Consider a wedge penetrating a collection of particles of average diameter D such that L/D = 1/5. The wedge encounters the equivalent of five particles on the average during an increment of penetration depth equal to L. At the moment of wedge-particle contact, a sudden rise in penetration load occurs; the particle moves and the load diminishes. The forces required for particle splitting and for particle acceleration as well are functions of particle diameter or size. Consequently, the amplitude fluctuations of penetration force about the mean penetration load will be a function of particle size as will be the frequency of such fluctuations. The mean force required for penetration of discontinua, however, is still amenable to a continuum-type of analysis. Discontinua do have strength in the aggregate that can be measured by test apparatus of sufficient size, and the utilization of this strength in a continuum type analysis leads to an estimate of the mean force-penetration function and the mean work-penetration function. The actual work-penetration function, however, is smooth after a penetration of several particle diameters. Smoothing of the actual work penetration function occurs through integration of the rapidly oscillating but steadily increasing force-penetration function.

Integration of such a curve is, in fact, self-smoothing because the fluctuations in the size of the work increment associated with the fluctuating force-penetration curve rapidly becomes small compared to the total work of penetration. Thus after a depth of several particle diameters, the work-penetration function becomes smooth. Differentiation of the smoothed work-penetration function for the discontinuum results in a force-penetration function of an "equivalent" continuum whose characteristics are, on the average, those of the parent discontinuum.

From the equipment design and selection standpoint it is obviously important to know the excursions from the mean penetration force as well as the mean, since in the case of discontinua, the excursions are large. The frequency of the excursions above the mean load which itself increases with depth indicates that the design load should be the upper limit of the force-penetration curve rather than the mean. Moreover, the force-penetration function will be preferable to the work penetration function as a basis for design-selection decisions because the digging force excursions above the mean are not revealed by the smoothed

work penetration curve. In the case of discontinua, it is not sufficient to design and select equipment on an "average" basis.

RESEARCH PLAN

This investigation is an extension of recent materials handling research at the Pittsburgh Mining and Safety Research Center, but with samples of materials obtained during typical rapid excavation operations. The first step is the measurement of those physical properties that are pertinent to the mechanics of flow and loading of the bulk material. The second step involves experiments dealing with the penetration and loading process, analysis of the data in terms of combinations of the physical properties, including usage of existing theory applied to penetration and excavation analysis, generation of empirical models from values of these parameters, and the formation of a finite element simulator.

RESULTS TO DATE

For the past year, the major efforts were to find the effects of the physical properties of granular mined material on penetration and excavation processes. The basic physical properties investigated include particle size, shape, moisture content, bulk density, angle of repose, angle of slide, specific gravity, Atterberg limits, angle of internal friction, angle of slide, cohesion, potential volume changes, and rock toughness (tables 2 and 3). The approach to the problem is to use these properties as parameters in the analysis of the experimental results. Equipment for this work basically consists of three large testing apparatuses; namely, a large direct shear tester, a linear penetration machine, and an excavation simulator.

TABLE 2. - Size Distribution
Sample WP-1

Size range		Percent weight of total	Percent weight finer
	+25.4	23.79	76.21
-25.4	+12.7	24.63	51.58
-12.7	+ 9.51	11.20	40.38
- 9.51	+ 4.76	10.08	29.7
- 4.76	+ 2.38	4.70	25.0
- 2.38	+ 1.19	1.97	23.03
- 1.19	+ 0.595	0.77	22.26
-0.595	+ 0.297	0.72	21.54
-0.297	+ 0.149	5.10	16.44
-0.149	+ 0.105	6.74	9.70
-0.105	+ 0.074	9.70	-

Thirty-eight samples have been received and prepared for this work. The samples represent different types of granular material which were collected from 15 mining and tunnelling sites representing both conventional drill and blasting and boring machine excavation methods.

During the measurement of the physical properties, certain phenomena have been detected. The size distribution of particles is generally more uniform and the particles more plate like when machine boring is used compared with the conventional drill and blast method. Water content does not show any significant effect on any of the properties measured. The angle of repose is dependent on the height from which the sample is released. The higher the distance, the lower the angle of repose. For the angle of slide, the material consisting of a higher percentage of fines has a higher value. Potential volume change is very insignificant, indicating little or no clay in the samples. The angle of internal friction and cohesion are directly affected by the particle size, shape, and hardness.

Experimental work has been completed on 15 samples for penetration tests and 7 samples for loading tests. For both penetration and loading tests, difficulty has been encountered when harder material was used. Modifications on penetration testing equipment have been finished, and the loading apparatus is being modified at the machine shop.

Coulomb's theory of passive earth pressure was used in obtaining a theoretical relationship for penetration. The method was developed as follows. Assume that a plane face of a wedge is pressed against a mass of granular material with a horizontal and inclined surface. The passive earth pressure, P, and weight of the mass, γ, were used to take moments about points laying on the straight line, L, which is at an angle of $45^{\circ} - \phi/2$ to the horizontal inclined surface. The resulting equation is:

$$P = \frac{1}{2} \gamma H^2 \mathrm{Tan}^2 \left(45^{\circ} + \frac{\phi}{2}\right)$$

Assume that the curved part of the section through the surface of failure consists of a logarithmic spiral with the equation

$$R = R_o e^{\delta \mathrm{Tan}\phi}$$

whose center is located on the line L. The equilibrium of the system requires that the moment of all the forces about the center of the spiral must be equal to zero. The resultant force P_R acting on the wedge can be obtained through the equation

$$P_R = \frac{1}{L_R} \sum_{1}^{N} M_N .$$

The theoretical curves were constructed through the use of a computer program.

At the present time, the elastic-plastic finite element computer program under development at the Pittsburgh Mining and Safety Research Center is capable of solving two-dimensional (plane strain) boundary value problems involving quasi-static, isothermal deformation of layered, initially stressed, gravitating, orthotropic media exhibiting either linear or non-linear yield functions. Boundary conditions may be of the friction or displacement type or mixed. Sequences of material aggregation (fill mode) and erosion (cut mode) can be simulated. The program is operational and will be used in theoretical analysis of penetration. The program consists of a main line and four subroutines. Subroutines are called from the main line only. Generation of element stiffness matrices is accomplished by one subroutine; assembly of the master stiffness matrix is accomplished by another subroutine. The unknown displacements are computed in a third subroutine. The total solutions to the elastic and elastic-plastic analyses are written by the fourth subroutine. The constant strain, three node triangular element is employed. A stable (in the sense of Drucker) elastic-plastic material is assumed, so that the constitutive equations are taken in incremental form. An extended Von Mises type of yield condition, either linear or quadratic, is also assumed, so that the yield surface is smooth everywhere but at the apex. Solution is by Gauss-Siedel iteration, and iteration continues until the sum of the absolute values of the residuals is less than a specified number. A monotonically decreasing sum of the residuals is indicative of convergence at exit time. A solution to a problem is essentially a printout of element stresses and strains, and nodal point displacements. The output of failed element information during the incremental application of load reveals the progression of the failure zone and delineates the boundary between the elastic and the elastic-plastic zones. Additional information, including the nodal point residuals is also listed. A large core storage is required for the program in its present form. This is the only limitation as to the number of elements, nodal points, and material type that can be incorporated into an analysis. Core storage requirements can be altered by simply changing the COMMON statements.

Future additions to the program will be made for the purpose of simplifying program useage and improving program efficiency. However, the basic structure of the program probably will remain the same.

Based on the investigations, the following conclusions can be drawn:

1. In the direct-shear tests it was found that the angle of internal friction, shear stress, and volumetric expansion or dilation increase as the particle size increases, shape becomes very irregular, and particle hardness increases. There are considerable interlocking forces which contribute to the shear stress and apparent cohesion although it is commonly believed that no cohesion can exist in most of the materials of this type. It is apparent that the failure plane which develops in the granular material has a great effect on the other particles above and below the failure plane. The fines can develop some effects on the shear strength when moisture content is high. The volume

expansion during the shearing process is directly proportional to its compacted state or void ratio (figures 1 and 2).

2. For friction tests when a sample of material is moved over the surface of a steel plate, the angle of friction and volumetric expansion have lower values than in the direct shear test. An abrasive action at the metal-particle interface was detected. After both the direct-shear and the frictional tests, there is a considerable quantity of fine particles present in the shear box. This shows that during the tests there is some grinding between the granular particles resulting from the nonuniformity in composition and shape of the test material.

3. Penetration force-displacement curves change with respect to the internal friction angle. The higher the internal friction angle, the higher the average penetration force required for a given displacement.

4. The excavation tests generally show that the energy required to push the wedge into the granular material is very high, but that for a lifting or loading action, the energy required is much lower. This is caused by material compacting under the wedge and building up tremendous pressure. This experimental work could lead to new wedge design which can release the pressure buildup by changing digging angles.

5. The physical property measurements of the fragmented materials are used as parameters to analyze the results of in-house research and as input for a company that has a contract which has as goals the matching of equipment to materials being handled and the correlation of bulk material properties to in situ rock properties. Typical physical property testing results are summarized in table 3.

6. Linear penetration tests were run using different wedge configurations. These were full-, half-, and conical wedges. Testing results clearly show the conical wedge with an angle of 30° required the least force for penetration and gave a much smoother penetration curve even for very irregular granular particles. For both full- and half-wedges, the total surface in contact with the material directly affects the total penetration force. An example testing result is shown in figure 3.

7. By using Coulomb's theory of passive earth pressure of ideal sand and the logarithmic spiral method, theoretical penetration and digging curves were constructed. These agreed with actual results for most samples (figures 4, 5, and 6).

TABLE 3. - Basic Physical Properties of Mined Material

Sample	Moisture content, percent	Specific gravity	Atterberg limits				Flow index	Toughness index	Potential volume change	Angle of repose Height of drop 1"	10"	Angle of slide, degree	Angle of internal friction, degree
			Liquid limit, percent	Plastic limit, percent	Shrinkage limit, percent	Plasticity index, percent				degree			
SF-1	13-17	2.86	17.75	16.19	13.94	1.56	5.8	0.27	0	38	33	36	42
WP-1	5-7	2.73	16.90	15.50	15.18	1.40	5.0	0.28	0	35	29	28	29
NAST-1	8-10	2.69	14.50	14.00	13.50	0.5	3.0	0.16	0	37	36	41	42
LAW-3	6-7	2.80	11.80	10.6	10.0	1.2	2.9	0.41	0	41	40	38	32
LAW-2	12-13	2.83	12.50	12.3	9.6	0.2	4.0	0.05	0	39	38	31	30
H-2	2-3	2.70	18.0	17.0	13.4	1.0	4.4	0.23	0	40	37	32	44
SF-2	13-15	3.02	31.5	26.8	21.5	4.7	7.6	0.61	0	38	36	30	27
LAW-4	7-9	2.73	20.2	20.0	13.5	0.2	4.7	0.05	0	42	34	37	28
MB-1	5-7	4.34	17.8	15.1	13.9	2.7	4.1	0.66	0	37	35	31	35
PHIL	7-10	2.57	24.0	23.3	22.7	0.7	4.0	0.17	0	39	37	40	30
WP-2	2-3	2.63	23.0	17.63	17.58	5.37	6.9	0.78	0	32	31	29	44

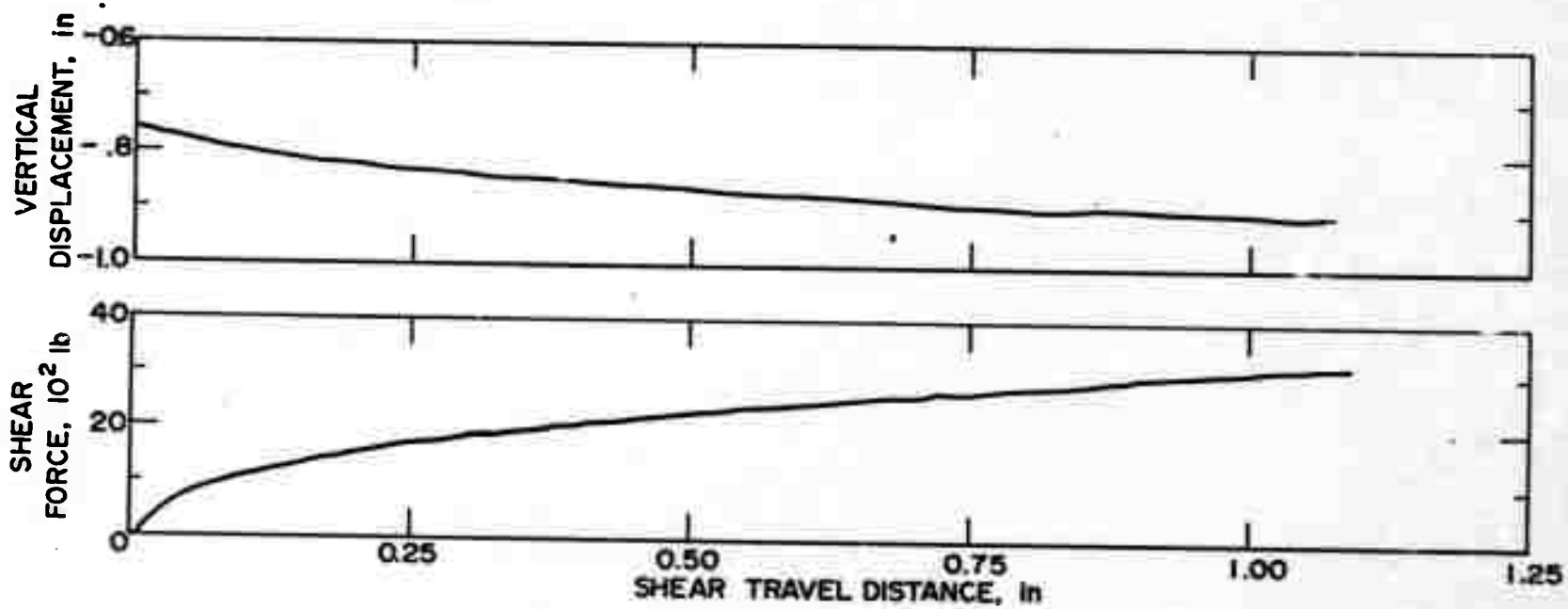

FIGURE 1. - Direct Shear Test Experimental Data

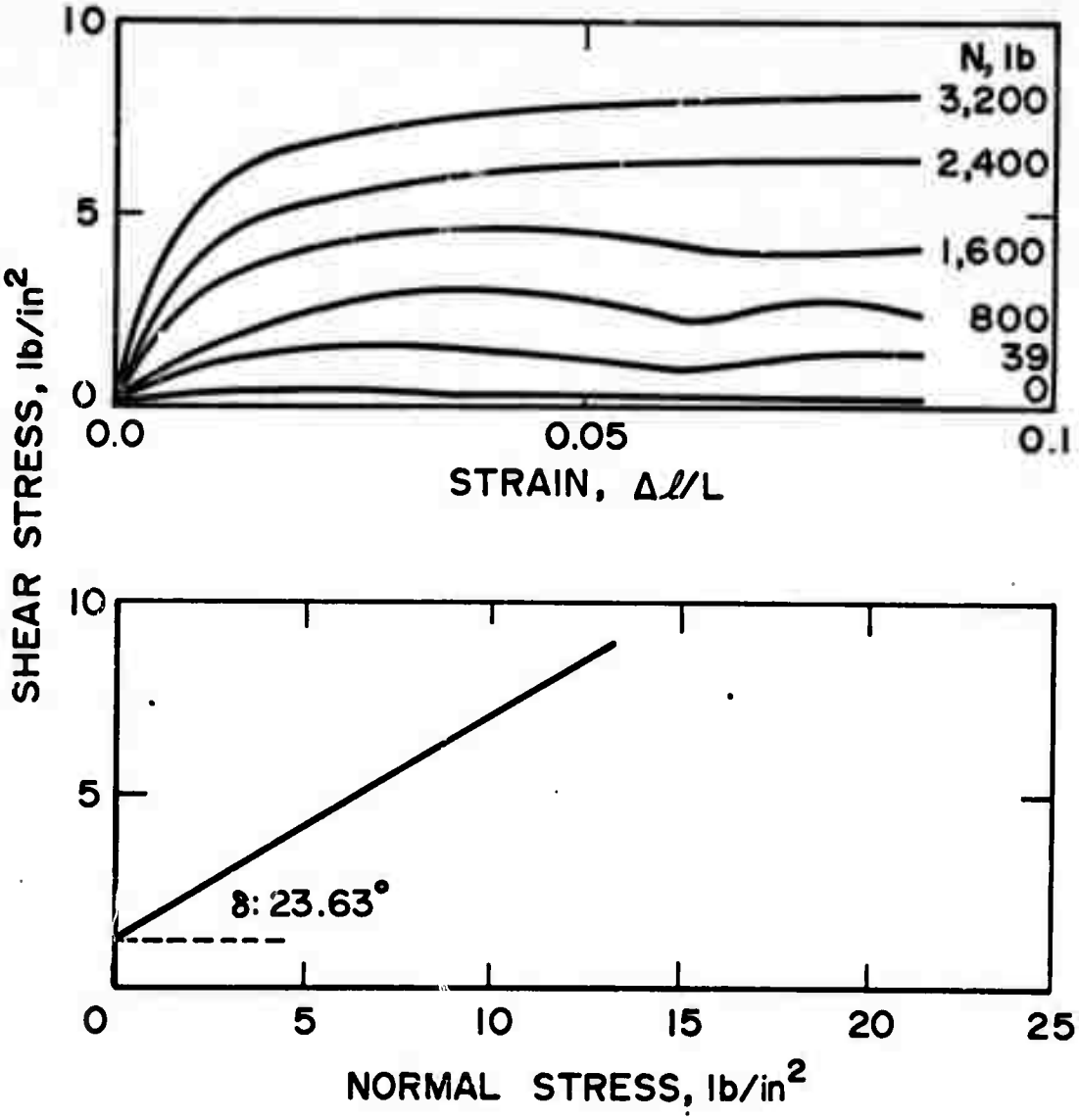

FIGURE 2. - Direct Shear Test Result and Stress-Strain Relationship

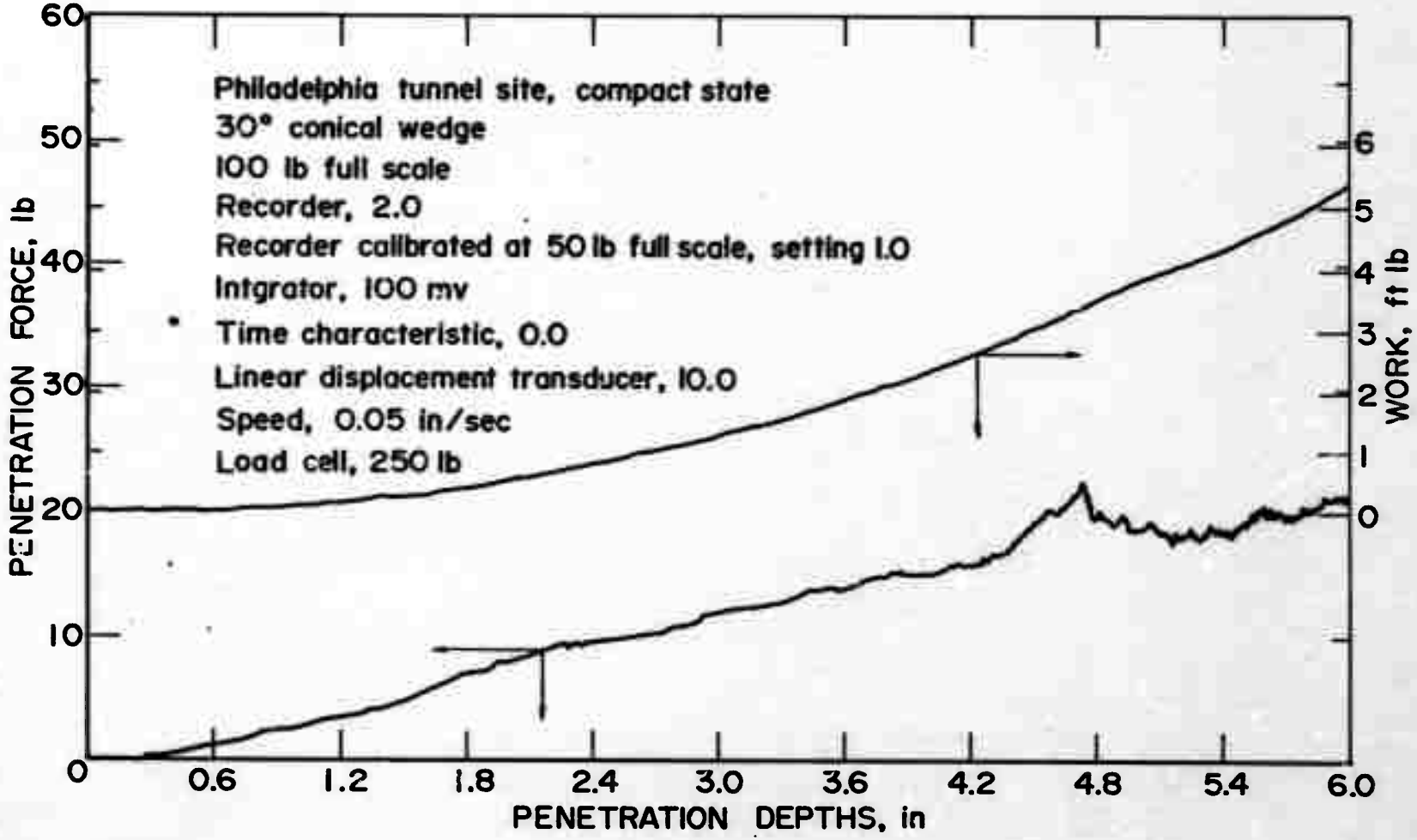

FIGURE 3. - Penetration Test Experimental Data

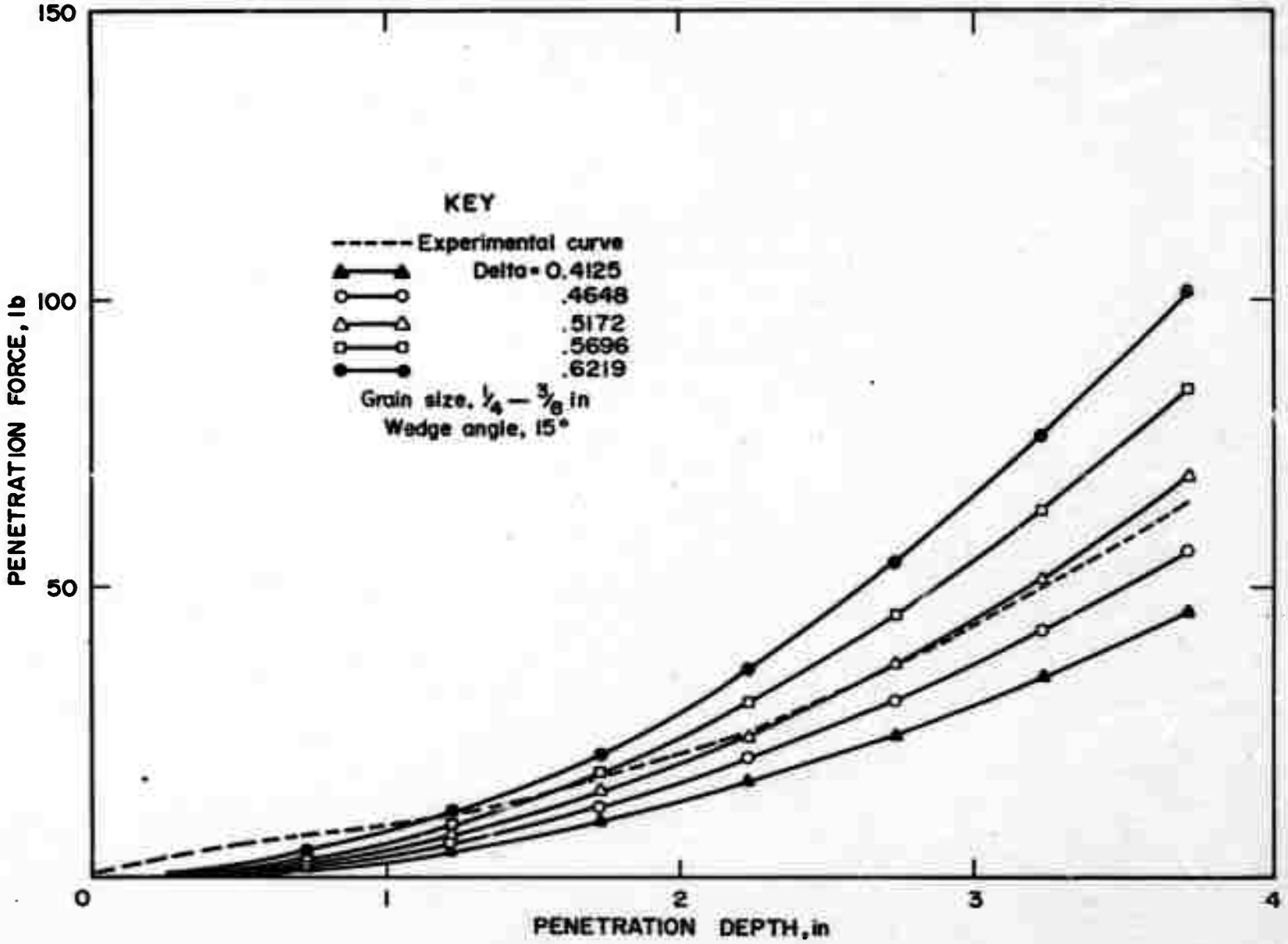

FIGURE 4. - Penetration Force Versus Depth

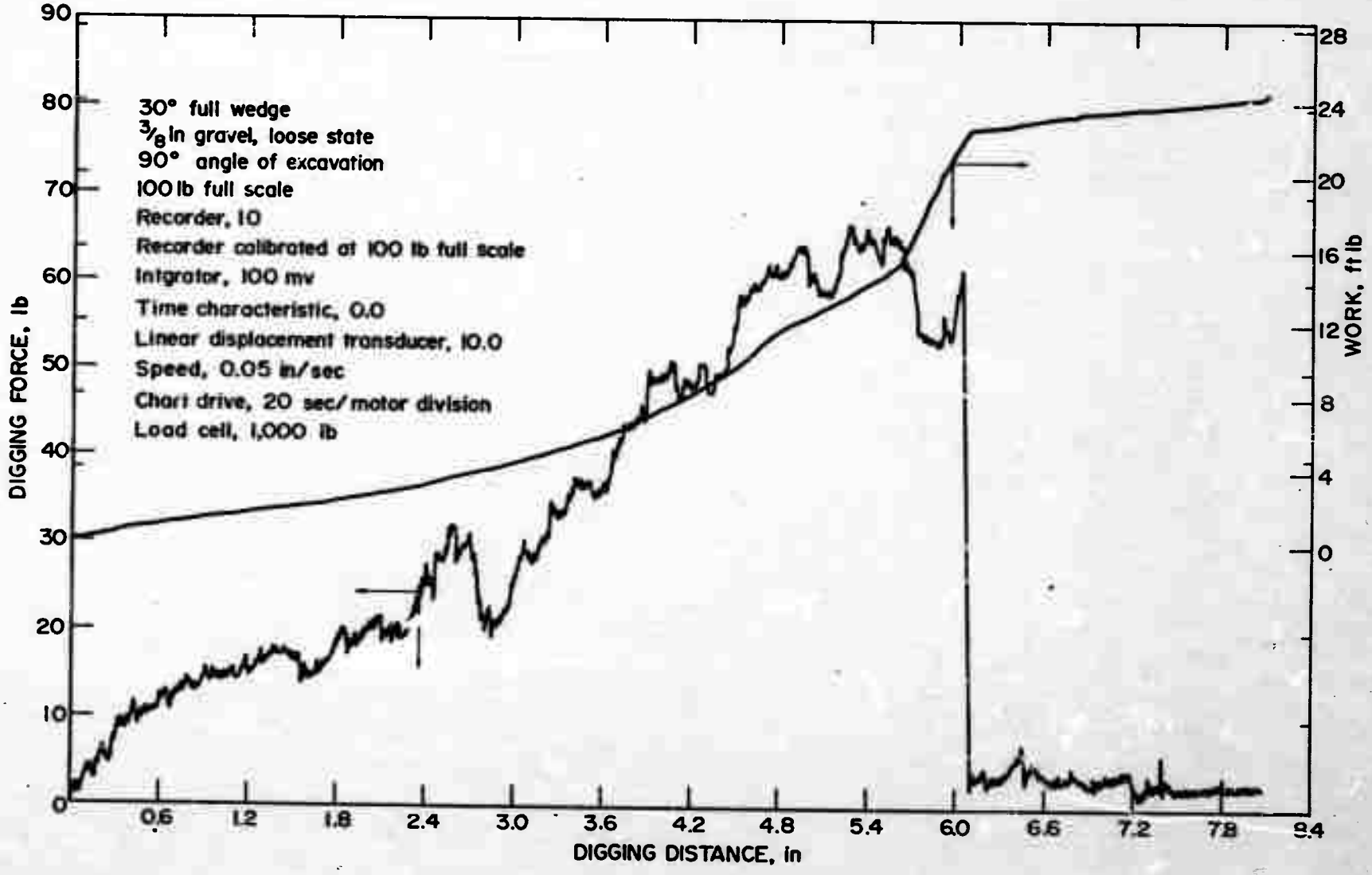

FIGURE 5. - Excavation Test Experimental Data

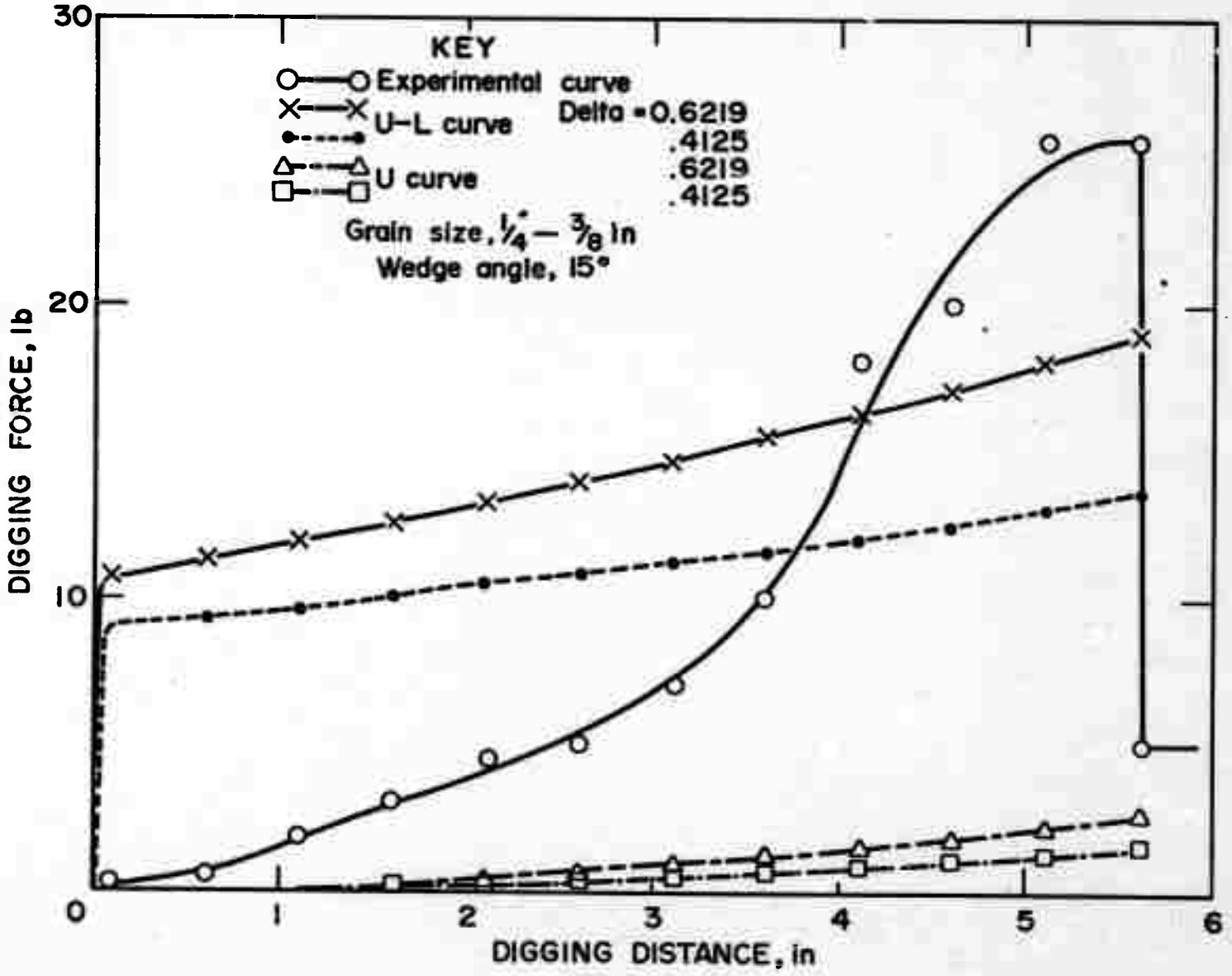

FIGURE 6. - Digging Force Versus Distance